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On the role of disorder in transport and magnetic properties of the two-dimensional electron gas

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Abstract

We describe the effects of disorder in a two-dimensional electron gas in the presence of a parallel magnetic field. We argue that localized states lead to the formation of local moments. The parallel magnetic field for complete spin polarization depends on the electron density and the density of local moments and we can explain the observed dependence of the saturation field of the magnetoresistance on the sample quality. The magnetic properties of the electron spins are described in terms of a Curie-like paramagnetism due to localized moments and a Pauli paramagnetism due to itinerant electrons.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

In recent years the transport properties of the two-dimensional electron gas (2DEG) have been studied in detail at low temperature. The experiments have been interpreted in terms of a metal–insulator transition (MIT) occurring at a critical electron density n_c [1]. Applying an in-plane magnetic field to the metallic phase of a diluted 2DEG leads to a strong positive magnetoresistance (MR), which saturates when the electron gas is fully spin polarized [2, 3]. The positive MR found for samples with weak disorder is in quantitative agreement with a calculation of [4], where the modifications of the screening properties and the density of states due to spin polarization are taken into account.

In this paper we describe the saturation field of the MR and the magnetic properties of a strongly disordered 2DEG when the system is in the metallic phase. We use a model developed earlier for the three-dimensional electron gas (3DEG) and confirmed experimentally [5–7]. We believe that this model has not been used before to explain transport properties of a 2DEG in a parallel magnetic field. We apply in this paper the term 'strongly disordered' to indicate that we consider a 2DEG in which the metallic phase is characterized by extended and localized electron states. With the term 'weak disorder' we describe a situation where the existence of localized states can be neglected.

The paper is organized as follows. In section 2 we describe the model and the theory. A short discussion is given in section 3. We compare our theoretical results with experiments in section 4 and we conclude in section 5.

2. Model and theory

Let us start with a dense 2DEG in which many-body effects (for instance exchange effects) can be omitted, e.g. $r_s \leqslant 1$ where r_s is the Wigner–Seitz parameter. Extension effects of the electron gas perpendicular to the surface are also neglected: orbital effects do not exist but spin effects are taken into account. An in-plane magnetic field leads to a Zeeman energy and to a partial spin polarization of the electron gas. In the 2DEG with weak disorder the spin-polarization parameter ξ is given as a function of the density of up and down electrons as $\xi = (n_u - n_d)/(n_u + n_d) = B/B_{c0}$. ξ controls the screening properties of the electron system at $B \leqslant B_{c0} = \varepsilon_F/g_b\mu_B$. Here g_b is the band value of the Landé g-factor, μ_B is the Bohr magneton and ε_F is the Fermi energy of the spin-polarized electron gas: $\varepsilon_F = n/\rho_F$ where ρ_F is the density of states of the spin-polarized 2DEG. For $B > B_{c0}$ the system is completely spin polarized; both screening properties and resistance are independent of the magnetic field. If $r_s \ll 1$ then, according to [4], the sample resistance decreases by a factor of two in the region $0 < B < B_{c0}$ with an increase of B. In systems with strong disorder localized states do exist even in the metallic phase deep under the Fermi level. The metallic phase is described as a system where $n > n_c$, which corresponds to $\varepsilon_F > \varepsilon_c$ (where ε_c is the mobility edge). Electron states with an energy $\varepsilon < \varepsilon_c$ are localized while electron states with an energy $\varepsilon_c < \varepsilon \leqslant \varepsilon_F$ are non-localized [8, 9].

In the absence of a magnetic field, extended states in metals have a spin degeneracy of two for spin-up and spin-down electrons. Due to the strong repulsive Coulomb interaction, localized states in the band tail can be singly occupied or doubly occupied: for weak disorder the density of singly occupied (so) localized states n_{so} will be small, $n_{so} \ll n_c$; for strong disorder all localized states are singly occupied, $n_{so} \simeq n_c$ [8].

Singly occupied states contribute to the thermodynamic properties of the 2DEG, despite the fact that they are placed deep below the Fermi level. The contribution can be considered in the framework of the phenomenological two-component model[3] for localized electrons (local moments) and extended (itinerant) electrons [5]. This two-component model was successfully used previously to describe thermodynamic and magnetic properties of the strongly disordered 3DEG as realized in phosphorus-doped silicon [5–7]. For spins without interaction, one assumes Curie paramagnetism for the localized singly occupied states of density n_{so} and Pauli paramagnetism for the states of density $n - n_{so}$. The singly occupied localized states are treated as classical spins and are spin polarized if the temperature T is sufficiently small: $k_B T \ll g_b\mu_B B_1$ where k_B is Boltzmann's constant. The electrons become completely spin polarized for $B > B_c$ with $g_b\mu_B B_c \leqslant (n - n_{so})/\rho_F$. B_c is expressed as

$$B_c = B_{c0} - \frac{2hn_{so}}{eg_v g_b m_b/m_e}, \tag{1}$$

where m_b is the band value of the electron effective mass and g_v is the valley degeneracy. The spin susceptibility χ is the sum of two terms:

$$\chi = \chi_{so} + \chi_s \tag{2}$$

[3] We remark that the term 'two-fluid model' was used in [5]. The local moments in this model are localized and we prefer to use the term 'two-component model' instead.

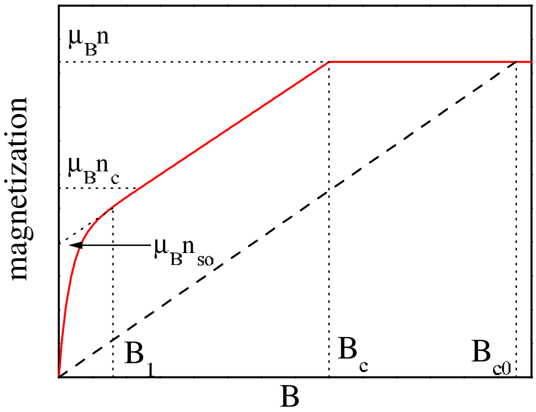

Figure 1. Magnetization as a function of the magnetic field B. The solid line represents the electron gas with localized and extended states within the two-component model. The dashed curve represents the electron gas without localized states. The fields B_1, B_c and B_{c0} are described in the text. We use for the effective g-factor $g^* = 2$.

due to localized moments (χ_{so}) and due to extended states[4] (χ_s). For non-interacting localized classical spins one can use the Curie law $\chi_{so} = n_{so}\mu_B^2/k_BT$. For interacting localized moments we can use numerical results from a model calculation for random Heisenberg antiferromagnets and the susceptibility is given by [10]

$$\chi_{so} = n_{so}\mu_B^2(T_0/T)^\alpha/k_BT_0; \tag{3}$$

$\alpha \simeq 0.75$ is an exponent and T_0 is a characteristic temperature. For $\alpha = 1$, one gets the Curie law. For itinerant electrons the spin susceptibility χ_s for low temperature is given in terms of the spin susceptibility of the free electron gas.

3. Discussion

In figure 1 we show what we expect to find for the magnetization of the disordered electron gas as a function of the magnetic field for $k_BT \ll \varepsilon_F$. For $B \ll B_1$ the magnetization increases linearly with the field. For $B \simeq B_1$ the localized spins are all oriented. For $B_1 < B$ the physics discussed above also applies: the localized electrons of the band tail contribute to the spin susceptibility, to the magnetization and to the specific heat. The magnetic field corresponding to the complete spin polarization and to MR saturation is expected to be smaller than in the case of a 2DEG with weak disorder. Nevertheless, the equations written above have to be modified and the applicability of these equations to the real situation has to be considered in more detail. For $r_s \gg 1$ the magnetization of delocalized electrons in figure 1 is not expected to be a linear function of the magnetic field any longer: many-body effects in the 2DEG give rise to an effective mass $m^*(n, \xi)$ and to an effective Landé factor $g^*(n, \xi)$.

In the theory for the MR for strong disorder [11], where localized states are present, the resistivity ratio $\rho(B > B_c)/\rho(B = 0)$ is strongly enhanced near the MIT compared to the resistivity ratio for weak disorder. Moreover, for $r_s \gg 1$ one finds a positive MR. The saturation field B_c was not calculated in [11]. We suggest that B_c is given by equation (1) where g_b and m_b (and g_b and m_b in B_{c0}) are replaced by $g^*(n, B_c)$ and $m^*(n, B_c)$ respectively.

[4] Within the model as we consider in this paper, diamagnetic contributions to the magnetization are absent. However, we expect in real structures, which have a finite width perpendicular to the surface, diamagnetic contributions to exist.

Now we address the question of the experimental situation for Si MOSFET. We start from weakly disordered 2DEG systems. The MR was studied in [12–14] and there are, apparently, two different ways to treat the experimental data.

The first way, used in [12, 13], is to scale the MR in relatively weak magnetic fields, where the spin polarization ξ is defined by $g^*(n, B = 0)$ and $m^*(n, B = 0)$. According to [15] a parallel magnetic field does not affect the effective g-factor and the effective mass up to a spin polarization of about $\sim$20%. In [12, 13] this interval was found to be larger than 20%. Using the scaling procedure to extract the product $g^*(n, B = 0)m^*(n, B = 0)$ gives values close to the results of direct measurements at low spin polarization [16].

The second way is to measure the magnetic field of the saturation of the MR [14]. Using this procedure the product $g^*(n, B_c)m^*(n, B_c)$ was determined. It is not very surprising that the results from these two treatments of experimental data are different, especially in the dilute limit.

Recently Pudalov *et al* [14, 17] measured the in-plane MR of (100) Si inversion layers and demonstrated that the magnetic field for MR saturation B_{sat} depends strongly on the sample quality. For given density the saturation field in experiments was found to decrease with increasing disorder. We believe that in these experiments the influence of singly occupied states on the saturation field is being observed.

We stress that localized electrons are not easily seen in transport measurements of samples which are in the metallic phase. For instance, the carrier density measured via Shubnikov–de Haas (SdH) oscillations in a quantizing magnetic field n_{SdH} is insensitive to the number of electrons in the band tail and gives the total number of electrons $n_{SdH} = n$. This fact has been known since the pioneering work [19], in which a Si MOSFET system was measured with a peak mobility of 3000 cm^2 V^{-1} s^{-1}, which is about an order of magnitude smaller than in the best modern samples. In this sample the number of electrons in the band tail (localized electrons) is estimated as 5×10^{11} cm^{-2} (the mobility vanishes at this density). Nevertheless, the SdH period is consistent with the density measured in a high magnetic field and the total number of Landau levels (or the oscillation number) is consistent with that calculated for the total number of electrons.

According to a number of experimental results [12, 14, 18], this density n_{SdH} is nearly identical to the density n_H measured using the Hall voltage. This fact has sometimes been interpreted as evidence for the absence of localized states in the metallic phase of the samples studied, despite the quite different n_c-values of the different samples. We stress that localized states are present in metallic samples. The conclusion that one must draw from the result $n_{SdH} \simeq n_H$ is that one cannot identify localized states by means of the Hall resistivity. It appears that the Hall resistivity is non-critical at the MIT.

In figure 2 we compare the experimental results of [14] for B_c with a simple fit based on the ideas developed above. We neglect the dependence of the effective mass and the effective g-factor on the spin polarization—it appears that the sample quality plays the most important role in determining the value of B_c. For the best sample (Si9Nj) with peak mobility $\mu_{peak} = 4.3$ m^2 V^{-1} s^{-1} a larger saturation field than that calculated in a such way was reported [14]; see figure 2. We believe that this could be caused by the above-mentioned dependence of the 2DEG parameters on ξ.

With n_{so} as a fitting parameter we find good agreement with experiment if we use $n_{so} = 0.6 \times 10^{11}$ cm^{-2} for sample Si22/9.5 and $n_{so} = 1.1 \times 10^{11}$ cm^{-2} for sample[5] Si43. The density dependence of the g-factor g^* and the effective mass m^* are taken into

[5] In three dimensions and for $n > n_c$ it was found that the density of local moments decreases with increasing electron density [6, 7]. We use for the fit in figure 2 a constant value of n_{so}, independent of the electron density and also independent of the magnetic field.

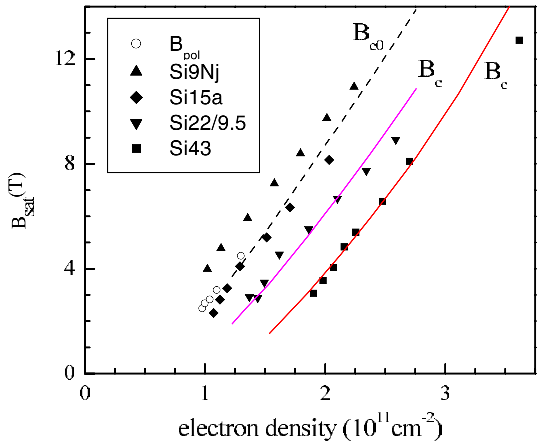

Figure 2. Saturation field B_{sat} as a function of the electron density n. The solid curves represent B_c according to equation (1). The dashed line shows B_{c0} for the clean system. The solid symbols represent the saturation field as determined from experimental MR data [14]. The empty circles represent B_{c0}, with g^* and m^* determined from very recent measurements of the SdH effect [17].

account by using experimental data [15, 20]. For sample Si43 the peak mobility was given as $\mu_{peak} = 1.96$ m^2 V^{-1} s^{-1} with $n_c = 1.4 \times 10^{11}$ cm^{-2} [21] and we conclude that $n_{so} = 0.8n_c$. For sample Si22/9.5a with $\mu_{peak} = 2.7$ m^2 V^{-1} s^{-1} we estimate n_c as $n_c \sim 1.2 \times 10^{11}$ cm^{-2} and get $n_{so} = 0.5n_c$. For the saturation field for sample Si15a with $\mu_{peak} = 3.8$ m^2 V^{-1} s^{-1} and $n_c = 0.82 \times 10^{11}$ cm^{-2} [21] we used B_{c0}. We find that for increasing peak mobility the critical density decreases in agreement with theory [22]. According to Mott [8], one expects $n_{so} \leqslant n_c$ for silicon MOSFETs with low peak mobility and $n_{so} \ll n_c$ for silicon MOSFETs with high peak mobility, in agreement with our results.

The results reported in this paper may have some impact on the understanding of ferromagnetism in disordered magnetic systems. A transition to a ferromagnetic state was found recently in diluted magnetic semiconductors such as $Ga_{1-x}Mn_xAs$ [23]. We believe that the density of singly occupied states n_{so} could play the role of magnetic ions, giving rise to a ferromagnetic state of the electron gas at low electron density.

In conclusion, we argue that according to our analysis, recent experiments [14] indicate local moment formation in the 2DEG in the metallic phase. We suggest that an experimental determination of B_c would be useful in order to obtain more information on the density of singly occupied localized states. Experiments should be extended to other 2DEGs and should include direct measurements of magnetic properties.

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References

[1] Abrahams E, Kravchenko S V and Sarachik M P 2001 *Rev. Mod. Phys.* **73** 251
[2] Okamoto T, Hosoya K, Kawaji S and Yagi A 1999 *Phys. Rev. Lett.* **82** 3875
Okamoto T, Hosoya K, Kawaji S, Yagi A, Yutani A and Shiraki Y 2000 *Physica* E **6** 260

[3] Vitkalov S A, Zheng H, Mertes K M, Sarachik M P and Klapwijk T M 2000 *Phys. Rev. Lett.* **85** 2164
[4] Dolgopolov V T and Gold A 2000 *Pis. Zh. Eksp. Teor. Fiz.* **71** 42 (Engl. transl. 2000 *JETP Lett.* **71** 27)
[5] Ue H and Maekawa S 1971 *Phys. Rev.* **3** 4232
Quirt J D and Marko J R 1971 *Phys. Rev. Lett.* **26** 318
[6] Paalanen M A, Graebner J E, Bhatt R N and Sachdev S 1988 *Phys. Rev. Lett.* **61** 597
[7] Lakner M, von Löhneysen H, Langenfeld A and Wölfle P 1994 *Phys. Rev.* B **50** 17 064
[8] Mott N F and Davis E A 1971 *Electronic Processes in Non-Crystalline Materials* (Oxford: Clarendon)
[9] Ando T, Fowler A B and Stern F 1982 *Rev. Mod. Phys.* **54** 437
[10] Bhatt R N and Lee P A 1981 *J. Appl. Phys.* **52** 1703
Bhatt R N and Lee P A 1982 *Phys. Rev. Lett.* **48** 344
[11] Gold A 2000 *Pis. Zh. Eksp. Teor. Fiz.* **72** 401 (Engl. transl. 2000 *JETP Lett.* **72** 274)
[12] Shashkin A A, Kravchenko S V, Dolgopolov V T and Klapwijk T M 2001 *Phys. Rev. Lett.* **87** 086801
[13] Vitkalov S A, Zheng H, Mertes K M, Sarachik M P and Klapwijk T M 2001 *Phys. Rev. Lett.* **87** 086401
[14] Pudalov V M, Brunthaler G, Prinz A and Bauer G 2002 *Phys. Rev. Lett.* **88** 076401
[15] Pudalov V M, Gershenson M E, Kojima H, Butch N, Dizhur E M, Brunthaler G, Prinz A and Bauer G 2001 *Preprint* cond-mat/0105081
[16] Kravchenko S V, Shashkin A A and Dolgopolov V T 2001 *Preprint* cond-mat/0108186
[17] Pudalov V M, Gershenson M E and Kojima H 2002 *Preprint* cond-mat/0201001
[18] Pudalov V M, Brunthaler G, Prinz A and Bauer G 1999 *Pis. Zh. Eksp. Teor. Fiz.* **70** 48 (Engl. transl. 1999 *JETP Lett.* **70** 48)
[19] Fowler A B, Fang F F, Howard W E and Stiles P J 1966 *Phys. Rev. Lett.* **16** 901
[20] Shashkin A A, Kravchenko S V, Dolgopolov V T and Klapwijk T M 2001 *Preprint* cond-mat/0111478
[21] Pudalov V M, Brunthaler G, Prinz A and Bauer G 1999 *Phys. Rev.* B **60** R2154
[22] Gold A 1985 *Phys. Rev. Lett.* **54** 1079
[23] Ohno H 1998 *Science* **282** 951